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Mapping the Magnetic Field at the Core-Mantle Boundary: Constraints on the Geodynamo

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ABSTRACT

Despite several centuries of conjecture and study, the details of the dynamo process in Earth's fluid outer core, which maintains the magnetic field against ohmic dissipation, remain elusive. Although recently there has been renewed optimism that numerical models might provide insight into the dynamo process, currently such models are severely restricted, even with the most powerful supercomputers. However, valuable insight can be gained from observations of Earth's magnetic field made at the surface, especially for time periods with sufficient density of observations to permit mapping of the field at the core-mantle boundary. From these observations, a time-dependent map of the magnetic field at the core-mantle boundary spanning the period 1690–1990 shows that the evolution of the field at the core-mantle boundary is highly variable, both spatially and temporally. To unravel this complicated signal it is necessary to proceed in stages. First, we examine the time-averaged, or steady, ingredient of the field. This part is nearly antisymmetric about the geographical equator and is dominated by concentrations of flux at high latitude in each hemisphere which account for the predominantly axial dipolar nature of the surface field. Surprisingly, the flux at the geographical poles, where an axial dipole field has maximum flux, is near zero. The remaining field—that is, the time-dependent part of the field left after removing this time-averaged field—is explained largely by a simple, steady-fluid-flow model at the core surface. This flow is almost symmetric about the geographic equator, a symmetry that is consistent with the antisymmetry of the steady field. Paleomagnetic data give further evidence of the importance of these symmetries in the dynamo process, both during intervals of normal secular variation of the field and during geomagnetic reversals. This simple field and flow morphology is consistent with a simple heuristic dynamo model based on the combined effects of Earth's rotation and the geometry of the outer core. A small unexplained signal remains after removing the effects of this steady flow. Part of this remaining signal is explained by time-dependent flow at the core surface. This flow can be used to calculate changes in the angular momentum of the core. The calculated changes agree well with observations of decadal variations in the length of day (rotation rate of the mantle), providing a clear demonstration that decadal variations in Earth's rate of rotation result from the exchange of angular momentum between the mantle and the core. Furthermore, this agreement provides an independent verification that time-dependent flow can be resolved, even though most of the field is explained by the steady-field model and the effects of steady flow.

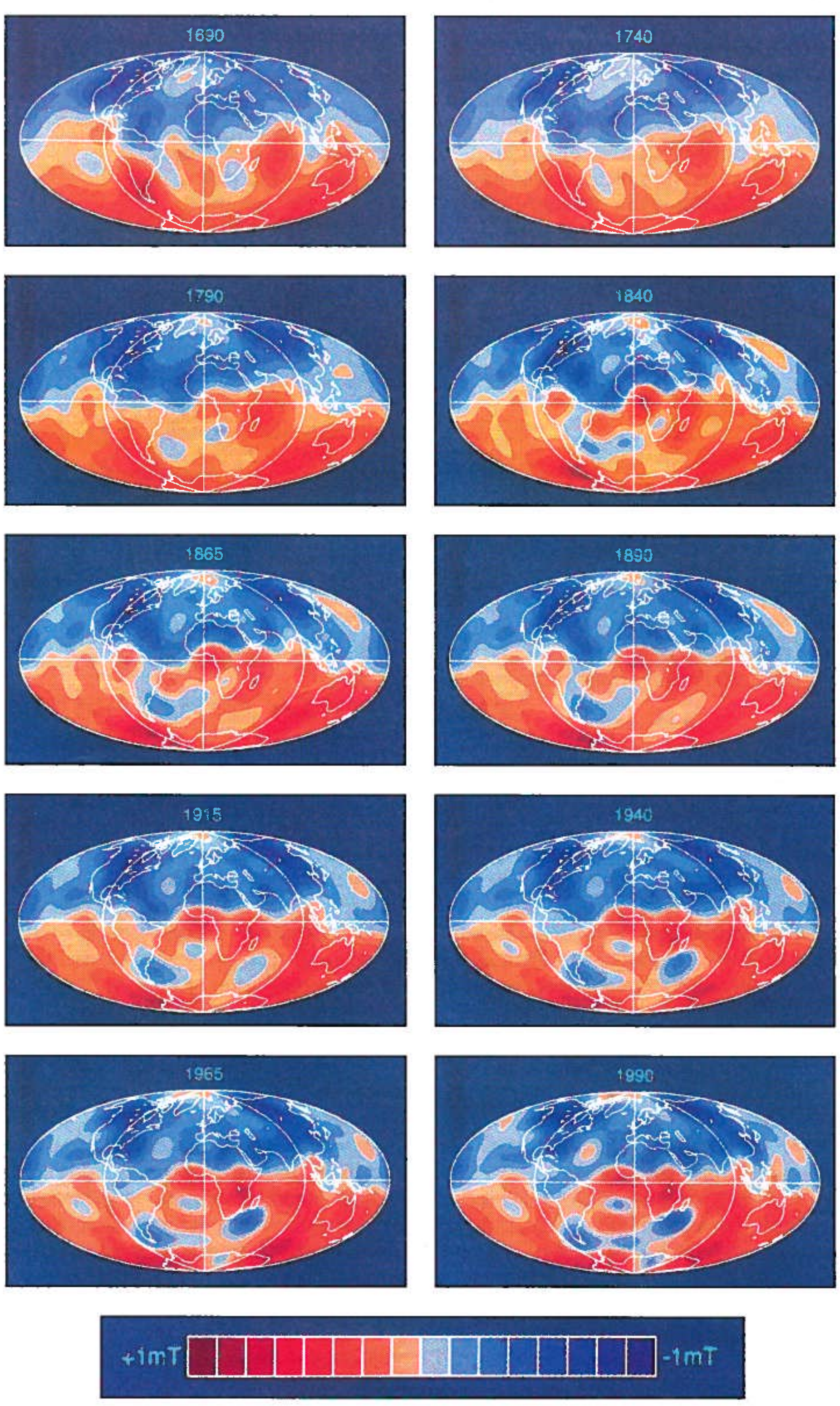

Figure 1. The radial component of the magnetic field at the core-mantle boundary calculated from models ufm1 and ufm2 of Bloxham and Jackson (1992). The contour interval is 100 μT. Hammer-Aitoff equal area projection.

INTRODUCTION

Understanding the mechanism responsible for the generation of Earth's magnetic field is one of the most longstanding problems in science; indeed, Albert Einstein once ranked it among the three most important unsolved problems in physics. The internal origin of the field has been recognized since the famous experiments of William Gilbert in 1600, but attempts at explaining its origin within Earth have encountered formidable obstacles. Although self-exciting dynamo action, originally suggested by Sir Joseph Larmor in 1919, is now widely accepted, it too once appeared to face insurmountable problems, given Cowling's (1934) demonstration that axisymmetric magnetic fields cannot be maintained by dynamo action. Cowling's result had an impact beyond the rather restrictive conditions of its applicability, in part because Earth's field is predominantly axisymmetric (at least at Earth's surface) but also because it was believed likely that his analysis was the first step in a more general antidynamo theorem. However, Backus (1958) and Herzenburg (1958) independently demonstrated that homogeneous self-exciting dynamos do exist, thus opening the way to the development of dynamo theory as an explanation of Earth's magnetic field. In the 35 years following their work, our understanding of the dynamo process has remained sketchy at best; such fundamental quantities as the strength of the magnetic field within the core are still unknown to within an order of magnitude.

The problem of understanding Earth's dynamo may be likened to attempts at understanding the dynamics of the atmosphere, but with several added difficulties: the magnetic field in the core plays an active role in determining the global circulation in the core, which adds considerably to the complexity of the governing equations; the core can only be observed indirectly, thus providing a much less immediate indication of the circula-

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Editor's Note:

Each year the David and Lucile Packard Foundation awards 20 Fellowships for Science and Engineering in a national competition to promising young scientists and engineers working in fields that receive relatively less popular attention than high-energy physics, space, and medicine. Each Packard Fellowship provides \$100,000 per year for five years to the Fellow's institution, \$90,000 of which is for use of the Fellow to support his/her research work. These young researchers are truly among the "best and brightest" in the United States. The science article in this issue is one of several in which Packard Fellows in earth science report on research in their field.

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tion; and because the time scale of the core flow is much longer than that of the atmosphere, observations of the core cover a relatively much shorter time span. I consider here how the rather limited observations that are available can be used to help constrain hypotheses of the dynamo process and, more generally, how they can be used as a probe of the structure and dynamics of Earth's deep interior.

OBSERVATIONS OF EARTH'S MAGNETIC FIELD

Direct observations of the magnetic field date back 500 years, and observations with sufficient geographical coverage to allow global mapping of the field date back more than 300 years. Although navigation has always been the overriding motivation for magnetic field observations, the importance of systematic observations of the field as a means to understanding its origin has long been recognized. At the end of the 17th century, Edmund Halley hoped that by refining his theory that the secular variation of the surface field can be described by a westward drift of contours of the magnetic declination (Halley, 1683, 1692), the origin of the field could be better understood. In this spirit, he undertook a magnetic survey of the Atlantic Ocean, perhaps the first marine geophysical survey.

Prior to the mid-19th century, most observations were made on sailing ships on trading voyages and voyages of discovery. The advantage of these observations is that they are generally not significantly contaminated by deviation, because they were made on wooden ships. However, imprecise navigation affects their accuracy; until the introduction of the marine chronometer by John Harrison in 1768, the determination of longitude at sea relied upon dead reckoning and was commonly in considerable error. Incidentally, Halley hoped that his magnetic survey of the Atlantic might also lead to a means of determining longitude at sea. In order to make use of these historical observations, we found it necessary to replot the original navigational records by relocating landfalls to their now known geographical positions and applying corrections to the intermediate positions.

Later, large-scale surveys of the field led, by the mid-19th century, to a great increase in the number of observations. Around this time, the first permanent magnetic observatories were introduced. Permanent magnetic observatories play a particularly important role because, unless relocated (an all too common occurrence), they are especially effective at constraining the time-varying part of the field.

Most recently, satellites, notably the POGO satellites in the middle to late 1960s and Magsat in 1979–1980, have provided very high quality observations with an extremely good geographical coverage.

All observations of the magnetic field are, of course, made at or slightly above Earth's surface. As a result, magnetic fields arising in the crust (and in the case of satellite measurements magnetic fields arising in the ionosphere) appear to be part of the internal field and so are indistinguishable from the core field. Fortunately, the crustal field is small compared to the core field at long wavelengths and to a large extent can be treated as a source of random noise, with an amplitude of a few hundred nanotesla. This is small compared to the amplitude of the field originating in the core, typically about

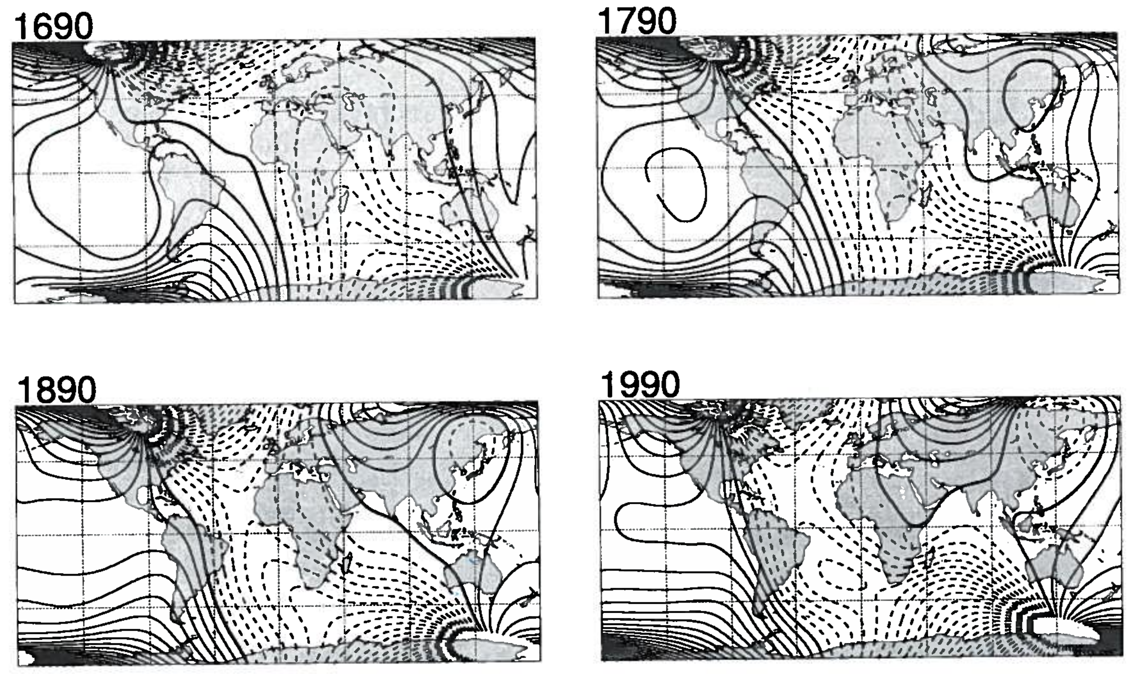

Figure 2. Magnetic declination at Earth's surface calculated from models ufm1 and ufm2. The contour interval is 5°; solid contours represent eastward declination, dashed-line contours represent westward declination, and the bold contour represents zero declination.

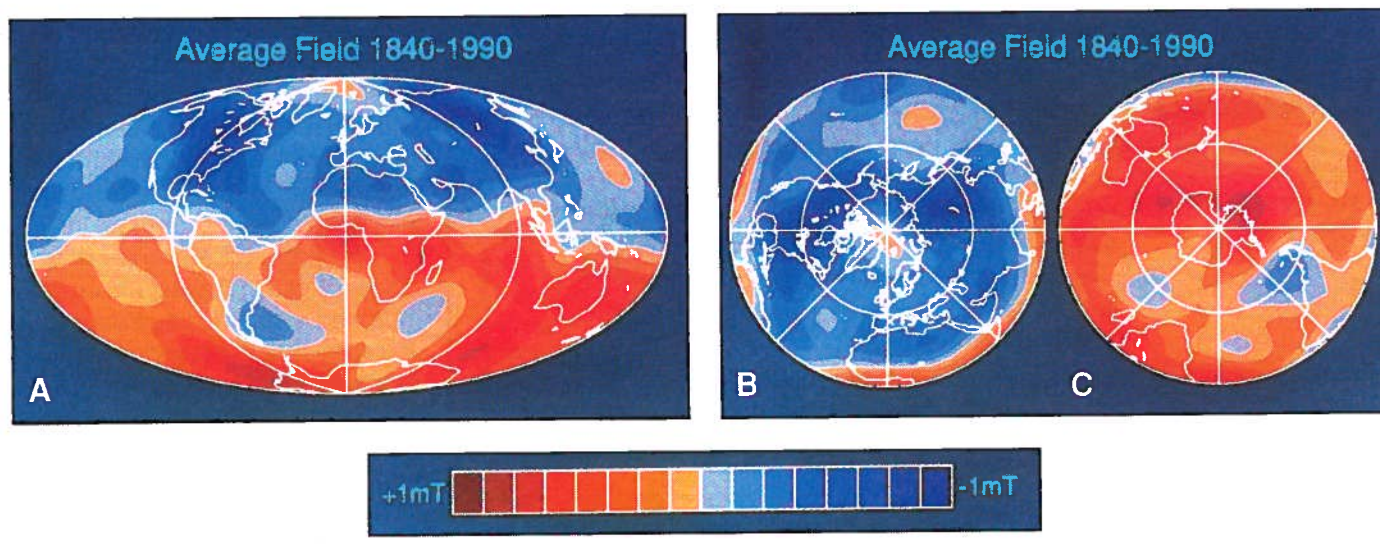

Figure 3. The radial component of the steady part of the magnetic field at the core-mantle boundary for the period 1840–1990, calculated from model ufm1. In A, the projection is Hammer-Aitoff equal area; in B and C, it is Lambert equal area centered on the north pole and south pole, respectively.

40 000 nT at Earth's surface. Ionospheric fields are even smaller in amplitude and are mostly of concern with recent high-resolution satellite observations. Statistical models of these contaminating fields can be constructed (Langel et al., 1989; Jackson, 1990) so that account can be taken when constructing field maps at the core-mantle boundary of their affect on the field originating in the core.

MAPPING THE MAGNETIC FIELD AT THE CORE-MANTLE BOUNDARY

This study is based on maps of the magnetic field at the core-mantle boundary rather than upon field maps at Earth's surface. One should like to be able to map the field within the core, but downward continuation is only possible through electrically weakly conducting regions (such as the mantle) and not into the body of the highly conducting outer core. Maps of the magnetic field at the core-mantle boundary do enable us to see, albeit incompletely, the magnetic field at the boundary of the dynamo region; as we shall see, maps at Earth's surface can be rather misleading when our aim is to understand the dynamo process in the core.

The discussion here is restricted to the radial component of the magnetic field for the simple reason that given the radial component, the horizontal components are uniquely determined and it is simpler to examine a single component of the vector field than all three components. The field is expanded using surface spherical harmonics to represent the spatial variations of the field and cubic B-splines to represent the temporal variations of the field; details were given by Bloxham and Jackson (1992). This expansion, which involves more than 14 000 parameters, is then fit to the observations by means of a method that finds the smoothest possible map for a given fit to the observations, so that any detail in the map is required in order to fit the observations to the particular choice of fit rather than being an artifact of the inversion procedure.

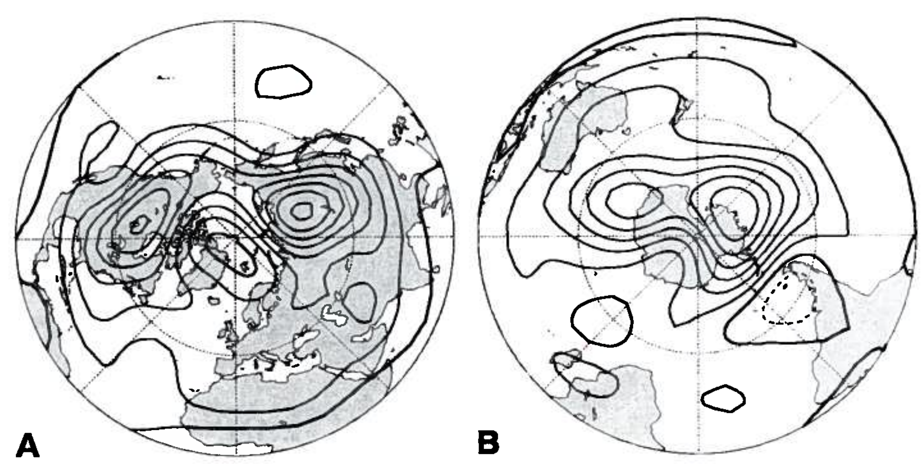

Figure 4. The function $B_r \cos \theta$ (see text) for the average field for the period 1840–1990 plotted using a Lambert equal-area projection centered on (A) the north pole and (B) the south pole.

The sequence of maps in Figure 1 shows the radial component of the magnetic field at the core-mantle boundary over the period 1690–1990, calculated from the field model of Bloxham and Jackson (1992). Although these maps are best viewed as a movie, it is clear nonetheless that the signal of the time-dependent magnetic field is rather complicated, and cannot, for example, be well described by a uniform westward drift. Beneath the Pacific Ocean the field exhibits no clear westward drift over this 300-year interval, or even any particularly obvious systematic time dependency. Figure 2 shows the declination at Earth's surface at 100-year intervals over the same period. In these maps westward drift is apparent, especially beneath the Atlantic Ocean, the region of Halley's study. This highlights the importance of mapping the field at the core-mantle boundary rather than at Earth's surface if our aim is to understand the origin of the field in the core; in particular, westward drift at Earth's surface should not be taken as an indication of a uniform westward drift of the outer core.

To move beyond the paradigm of westward drift, a more systematic analysis of the magnetic field at the core-mantle boundary is required. What is the best approach to unraveling the complicated signal of the time-dependent magnetic field portrayed in Figure 1? The analysis begins with the simplest possible first step: it is clear from Figure 1 that a large part of the field is unchanged over the time period considered, so we can begin by extracting the steady ingredient of the field—in other words, the time-averaged or

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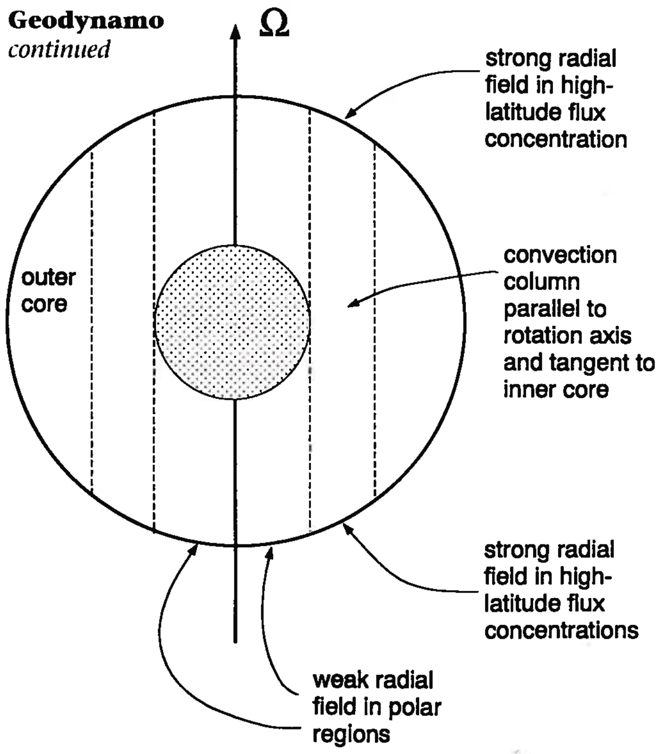

Figure 5. A model of the dynamo process in Earth's core. Dynamo action is concentrated in tall helical columns parallel to the rotation axis, the ends of which correspond to the concentrations of magnetic flux found at high latitudes. The polar regions of the core-mantle boundary, above and beneath the inner core, are regions of low magnetic field, possibly due to reduced dynamo action.

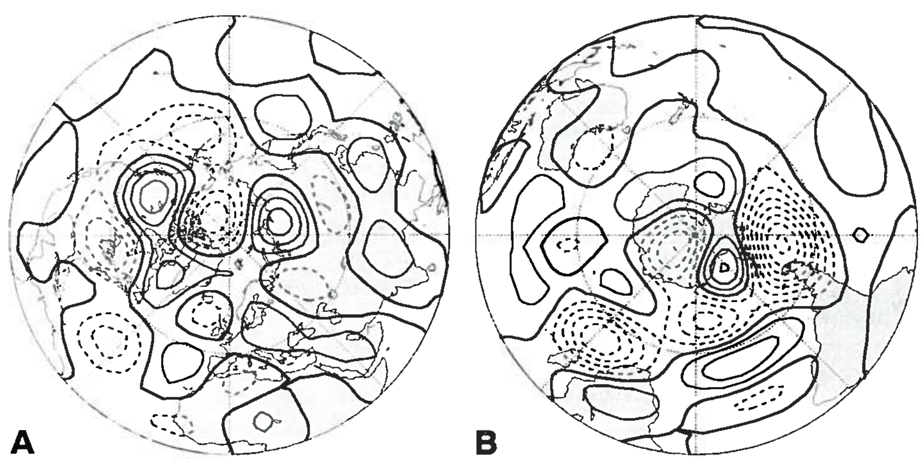

Figure 6. The function $B_r \cos\theta(t = 1990) - B_r \cos\theta(t = 1840)$ for the period 1840–1990 plotted using a Lambert equal-area projection centered on (A) the north pole and (B) the south pole.

non-time-dependent part of the field. This ingredient of the field is, most likely, the part that is best resolved, so given that we do not have reliable estimates of the true uncertainty of the maps, a conservative approach is to look first at this ingredient.

Steady Ingredient of the Field

The steady ingredient of the radial component of the field for the interval 1840–1990 (Fig. 3) accounts for approximately 80% of the variance of the radial field at the core-mantle boundary.

The dipole component of the field is due largely to two concentrations of flux in each hemisphere at high latitude and at approximately 120° longitude. The dominant contribution of these flux concentrations to the dipole is clearly seen in Figure 4, in which is plotted, using a polar projection, the contribution of the field to the axial dipole component, the quantity $B_r \cos\theta$ where B_r is the radial component of the magnetic field, and θ is colatitude (Gubbins, 1987). From Figure 1 it can be seen that these flux concentrations are nearly static. Closer to the poles the radial field is weak, and it is almost zero at the north pole, contrary to what would be expected for an axial dipole field, which would have maximum radial field at the poles.

As pointed out by Gubbins and Bloxham (1987), the steady field, including these high-latitude flux concentrations, is predominantly antisymmetric about the geographical equator, a permissible symmetry of the full magnetohydrodynamic dynamo equations, provided that the flow is symmetric about the geographical equator. This required flow symmetry is not unexpected, because of the strong role played by Earth's rotation in the dynamics of the core. An effect of rotation is to modify the form of the flow so that it is independent of the coordinate parallel to the rotation axis, resulting in columnar flow structures. But how do we explain the displacement of the high-latitude flux concentrations from the poles? Figure 5 shows one possible mechanism to explain this, on the basis of presence of the inner core. The columnar structures in the outer core are displaced from the rotation axis by the presence of the inner core, leading to flux concentrations at high latitude at the ends of these columnar structures and weak fields in the extreme polar regions within the cylinder tangent to the inner core.

Secular Variation

The next step in this analysis of the magnetic field at the core-mantle boundary is to look at the time-varying part of the field, the secular variation, which, mentioned above, varies greatly in strength over the boundary. We begin by considering the most prominent changes in the field.

The region of the core-mantle boundary that exhibits the greatest change in the field is that beneath southern Africa and the south Atlantic Ocean. In particular, a large patch of reversed flux (flux having opposite sign to that expected for that hemisphere with the present dipole polarity) has emerged and intensified in this region. What effect has the growth of this patch had on the axial dipole component of the field, which has decreased in strength by almost 10% in the past 150 years? From Figure 6, a plot of the change in $B_r \cos\theta$ (the quantity shown in Fig. 4) between 1840 and 1990 (i.e., the change in the contribution of the field to the dipole strength), we can see that the growth of the reversed flux patch and the decay of the dipole are closely related.

A simple model that explains the growth of this reverse flux patch is flux expulsion, the expulsion of initially horizontal field lines from the core by a concentrated upwelling motion (Bloxham, 1986). Remarkably, Bullard (1954) first proposed this mechanism to explain the anomalously large secular variation observed at southern African observatories, even though he had no maps available of the core field. Kinematically, this process is similar to that responsible for the formation of sunspots; by analogy, the resulting core field feature is called a core spot.

Although it is possible to account for particular features in the secular variation with simple models of this type (see Bloxham and Gubbins, 1985), to understand the secular variation globally rather than locally the magnetic field and secular variation are used to deduce maps of the fluid flow at the core surface. The secular variation of the magnetic field is described by the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \wedge (\mathbf{u} \wedge \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

which shows how the secular variation arises from advection of the field by the flow (the first term on the right-hand sides where $\mathbf{u}$ is the fluid-flow field) and from magnetic diffusion (the second term on the right-hand side, where η is the magnetic diffusivity). In the so-called frozen flux approximation, the effects of diffusion are neglected, so that the secular variation arises entirely from the distortion of field lines by the flow. In the absence of diffusion, field lines are effectively frozen in the fluid and so act as tracers of the flow. This equation can be inverted to determine the flow at the core surface, as first

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done by Kahle et al. (1967). However, in doing so, mathematical difficulties are encountered, as first elucidated by Roberts and Scott (1965) and Backus (1968). Under certain conditions, these difficulties may be at least partially circumvented (Voorhies and Backus, 1985; Backus and LeMouël, 1986; Lloyd and Gubbins, 1990; Jackson and Bloxham, 1991).

We seek the steady flow that best explains the secular variation. The rationale behind this is similar to that invoked earlier: the resolution of the core flow is uncertain, so a conservative approach is to seek first the steady part, because this should be the part that is best resolved. Figure 7 shows the steady flow for the period 1840–1990. This flow accounts for more than 98% of the variance of the secular variation. Note that westward flow is largely restricted to a region beneath the equatorial Atlantic Ocean, and there is no clear indication of westward drift elsewhere. The typical flow speed is about 15 km/yr or 0.5 mm/s. The flow is characterized by two large counter-rotating gyres, centered at mid-latitude in each hemisphere and at around 0° longitude. In Figure 7, the southern of the two gyres is more prominent. Beneath the Pacific, where secular variation is weak, the flow is also weak.

The flow is dominated by modes that are symmetric about the equator, the same flow symmetry mentioned above as necessary to preserve the symmetry of the steady part of the field. This is encouraging because not only does it simplify the development of dynamo models, but it also suggests that the deduced flow may indeed be indicative of the dynamo process in the core, rather than being merely some surface flow that bears little resemblance to the flow at depth in the core. An analogous problem exists in studying mantle convection: to what extent are surface plate motions indicative of the form of mantle convection? The motions at Earth's surface are obviously strongly influenced by rheological variations at the surface; similarly, core surface motions are influenced by conditions at the core surface. Among the many influences on core surface flow, lateral temperature variations in the mantle, which are most likely at least 10^4 times larger than those in the core (Stevenson, 1987), may be the most important. The effect of these mantle lateral temperature variations is to drive a thermal wind in the core (Bloxham and Gubbins, 1987; Bloxham and Jackson, 1990; Kohler and Stevenson, 1990). Although some evidence of this effect has been observed in the magnetic field (Bloxham and Gubbins, 1987), a challenge, in which some recent progress has been made by Zhang and Gubbins (1992), is to develop a more complete understanding of the interaction between this thermal wind and the underlying core convection.

Nonsteady Secular Variation

Because the steady part of the field accounts for over 80% of the variance of the field and steady flow accounts for over 95% of the remainder of the variance, only a very small proportion of the field model remains unexplained. Several arguments (Bloxham, 1992) suggest that this remaining signal is not entirely dominated by noise. This is remarkable because it suggests that the signal/noise ratio of the field maps is very much higher than might initially have been expected.

The strongest argument that the remaining signal is not entirely noise comes from the budget of angular momentum in the coupled core-mantle system. It has long been believed that, on the decadal time scale, variations in length of day (LOD) involve the exchange of angular momentum between the core and the mantle. The changes in angular momentum of the mantle are easily calculated from the LOD; the changes in angular momentum of the core require that we know, throughout the core, the time variations of those parts of the core flow that contribute to the axial angular momentum. Jault et al. (1988) have shown how these parts of the flow within the core can be estimated from the flow at the core surface. Although the assumptions that must be made about the core flow are quite strong (but not unrelated to the underlying dynamics) they found an encouraging agreement between the mantle and core changes in angular momentum over the period 1969–1985. Recently this has been corroborated over the longer time interval of 1840–1990 by Jackson et al. (1993) (see Fig. 8). This is an exciting result because it not only confirms the origin of decadal variations in the LOD, but it also provides an independent verification of the magnetic field and core surface flow models, because this agreement depends entirely upon this very small signal remaining after extracting the steady field and the steady secular variation.

Several interesting problems emerge from this study. First, magnetic signals from Earth's core are delayed on their passage through the mantle because of the electrical conductivity of the mantle. Some studies have suggested that this delay is as much as 13 years (Backus, 1983) or 18 years (Paulus and Stix, 1989). If this were the case, then we should see a lag, with the observed LOD leading the predictions based on the core flow; the lack of a discernible lag of this form in Figure 8 provides an upper bound on the electrical conductance of the mantle. In fact, before 1900 a lag of the opposite sign is discernible; the origin of this lag, which is noncausal, is being investigated. A second problem is to understand the mechanism of this angular momentum exchange. The necessary core-mantle coupling is not only of interest in explaining the LOD but is also of great importance in dynamo theory. A third problem is to examine whether this agreement in the LOD can be extended to shorter periods—say, a few years. At these periods changes in the LOD also arise from the exchange of angular momentum between the mantle and the atmosphere and oceans, raising the possibility of studying the response of the core to very long period coupled ocean-atmosphere oscillations.

Comments

The period for which it is possible to map the magnetic field at the core-mantle boundary is comparable in length to the advective time scale in the core. Although we can draw inferences about the operation of the dynamo, such as the role of core-mantle interactions and field symmetries, our conclusions must necessarily be guarded. The dynamo almost certainly evolves on a much slower diffusive time scale (perhaps as long as 15 ka), rather than on this fast advective time scale. To study the field on the diffusive time scale, we must resort to paleomagnetic observations. Such observations are too sparsely distributed to permit field mapping of the sort described here, but they do provide a means of testing, over longer time scales, many of the suggestions I have described here.

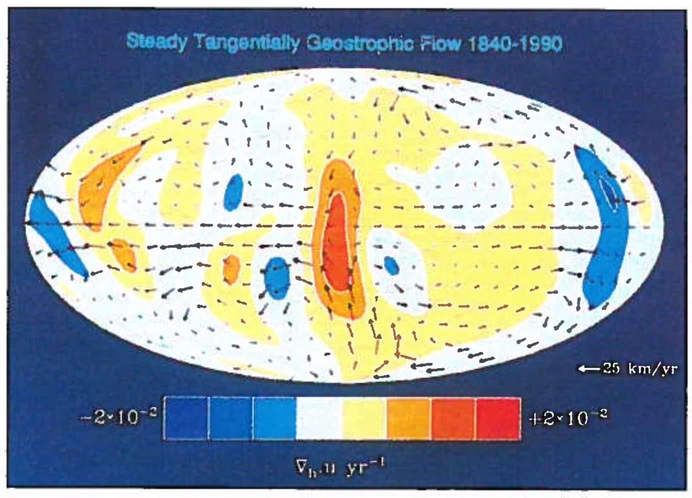

Figure 7. Map of the steady part of the fluid flow at the core surface from Bloxham (1992). The flow is constrained to be tangentially geostrophic (LeMouël, 1984). The vectors show the speed and direction of the flow at the core surface, and the scale shows the intensity of horizontal divergence (upwelling and downwelling) of the flow.

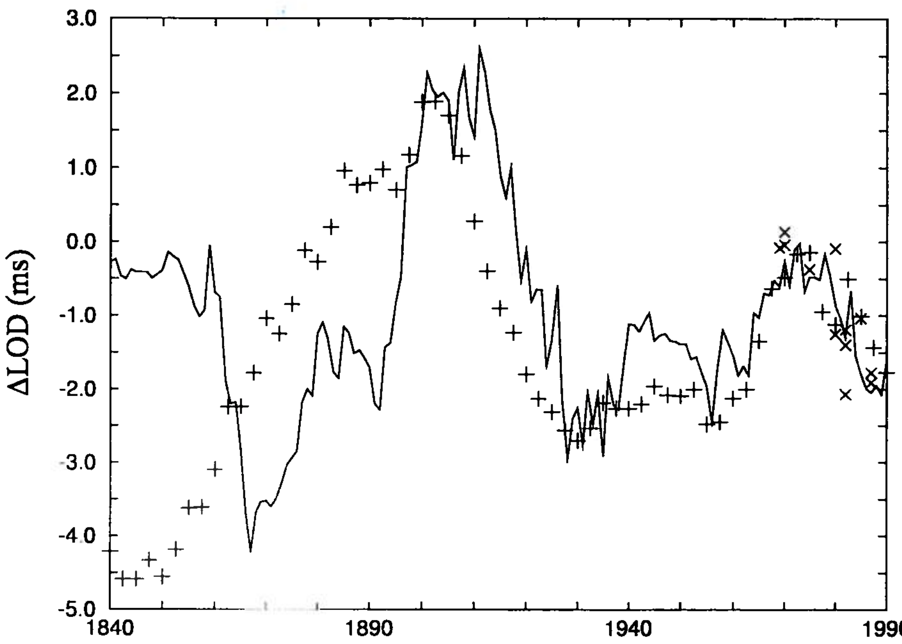

Figure 8. Comparison between the observed length of day (LOD; solid line) and the predictions based on models of the time-dependent part of the flow in the core. Plus-sign data are from Jackson et al. (1993); X-symbol data are from Jault et al. (1988).

Some significant progress has been made in this direction: Gubbins (1988) examined paleomagnetic data from the past 5 m.y. and found evidence of systematic departures from the axial dipole field consistent with the steady field configuration in Figure 3; Merrill and McFadden (1990) found evidence of the field symmetry property described above in the paleomagnetic secular variation; and, most recently, studies of geomagnetic reversals have indicated a tendency for the field during a reversal to exhibit aspects of the present-day field, notably a tendency for reversal paths to follow the longitude bands corresponding to the high-latitude flux concentrations identified in the recent core field (Clement, 1991; Laj et al., 1991; Constable, 1992). These are exciting results, which suggest that the historical field does provide an insight into the operation of the dynamo; furthermore, they suggest that aspects of the field morphology persist over time scales of millions of years, almost certainly the result of the influence of the mantle on convection in the core.

The highest priority in geomagnetism, though, is to ensure that this remarkable record of magnetic field observations is continued, preferably by further satellite missions. With continuous satellite monitoring, our ability to resolve both spatial and temporal detail in the field will improve dramatically, leading to greatly increased resolution of the time-dependent part of the flow and a more complete understanding of core-mantle interactions. Then we might at last be able to develop a working understanding of the dynamo. It is highly unfortunate that in the decade since Magsat our monitoring of the magnetic field has declined toward a level comparable to that of a century ago, and that the opportunity to build on the success of Magsat is slipping away.

ACKNOWLEDGMENTS

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